Name:_____ Date:_____

Math 9H HW 1.3 Prime Factorization, Perfect Squares, and Cubes (Calculators not Allowed)

1. Find the Prime factorization for each of the following numbers:

a) 24 d) 360 g) 9360

b) 17640 e) 9492 h) 9418409

2. Indicate which of the following numbers are perfect squares, perfect cubes, or neither:

a) $N = 16 \times 3^3 \times 14$ e) $N = 8 \times 25 \times 81$

b) $N = 4^3 \times 9^3$ f) $N = 7 \times 7^2 \times 7^4$

c) $N = 3 \times 4^3 \times 12$ g) $N = 27 \times 5^3 \times 135$

3. Simplify each of the following expressions without a calculator:

i) $\sqrt{12 \times 24}$ ii) $\sqrt{48 \times 243}$ iii) $\sqrt{32 \times 45 \times 8}$

iv) $\sqrt{6 \times 9 \times 24}$ v) $\sqrt{252 \times 567}$ vi) $\sqrt{147 \times 27 \times 289}$

vii) $\sqrt[3]{432 \times 375}$ viii) $\sqrt[3]{6 \times 60 \times 75}$ ix) $\sqrt[4]{252 \times 294 \times 42}$

4. Given that "N" is an integer, find the lowest value of N such that the following will be a positive integer:

a) $\sqrt{2^3 5^1 7^2 N}$ _____

b) $\sqrt{4^2 7^2 5^2 N}$ _____

c) $\sqrt{3^4 5^3 12N}$ _____

d) $\sqrt{38412N}$ _____

e) $\sqrt{13992N}$ _____

f) $\sqrt{664(N-1)}$ _____

g) $\sqrt[3]{2^3 11^3 49^2 N}$ _____

h) $\sqrt[3]{6^5 5^2 15^2 N}$ _____

i) $\sqrt[3]{3^4 5^3 28N}$ _____

5. Given that N is an integer and $N \neq 0$, what is the lowest value of N so that "K" is
i) a perfect square ii) a perfect cube iii) Both a perfect square and perfect cube $(Given\ K \neq 0)$

a) $K = N \times 3^3 \times 21$

b) $K = N \times 3^3 \times 5^5$

c) $K = N \times 75 \times 169$

d) $K = (N-1) \times 7^7 \times 121$

e) $K = N^2 + N$

f) $K = (3N-24)(N-28)$

6. Find the value of "N" in each equation:

$18,000 = N \times 2^3 \times 5^3 \times 6$

b) $80,640 = N \times 2^5 \times 12 \times 14$

7. For what integer $n > 0$ does $(100k)^2 \times (100k)^2 = n^2k^4$ for all values of k?

8. Let a, b, c, d, and e be distinct integers such that: $(6-a)(6-b)(6-c)(6-d)(6-e) = 45$. What is the value of $a+b+c+d+e$?

9. The number 1000 has 16 positive integral divisors. How many positive divisors does the number 3000 have?

10. Solve for 'n" $(n-23)!\times 23! = (n-32)!32!$

11. If "N" is the product of three different primes, then its least possible value is $2\times 3\times 5 = 30.$ If $N < 100$, what is N's largest possible value?

12. Challenge: Find the smallest value for N, such that N>1, so that the following expression is a perfect cube:
$5N^3 + N^2 + 15N$